MHD Effect on Free Convection Heat Transfer in a C-shaped Cavity

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Abstract

MHD effect on free convection heat transfer in a C-shaped cavity has been numerically simulated in this paper. The working fluid is assigned as air with a Prandtl number of 0.71 throughout the simulation. The magnetic field of strength B_o is applied parallel to x-axis. The top, left and bottom walls of the enclosure are maintained at constant heated temperatureT_h. The right internal walls of the cavity with length H are maintained at cold temperature T_c. The external right walls are adiabatic. The governing differential equations (mass, momentum and energy equation) are solved by finite element method (Galerkin weighted residual method). The present results are validated by favorable comparisons with previously published results. The results of the problem are presented in graphical and tabular forms and discussed. The relevant parameters in the present study are Rayleigh numbers, Hartmann numbers and Prandtl number. Results are presented in the form of streamlines, isotherms, Local Nusselt number, velocity, temperature and average temperature for different parameters. The numerical results indicate the strong influence of the mentioned parameters on the flow structure and heat transfer as well as temperature. An optimum combination of the governing parameters would result in higher heat transfer. Moreover, it is observed that both the Rayleigh numbers and the Hartmann numbers significantly impact on the flow and thermal fields in enclosure.

Keywords: *Free convection, MHD, C-shaped cavity, FEM.*

NOMENCLATURES

g Gravitational Acceleration	X, Y dimensionless Cartesian Coordinates
Nu Nusselt Number	
P dimensionless Pressure	GREEK SYMBOLS
p Pressure	α Thermal Diffusivity
Pr Prandtl Number	β thermal expansion coefficient
Ra Rayleigh Number	ν KineAmatic viscosity of the fluid
Ha Hartman Number	θ dimentionless temperature
Re Reynolds Number	ρ Density of the fluid
T Dimensional Temperature	ψ dimensionless stream function
u, v x and y Cartesian velocity components	Subscripts
U, V X and Y dimensionless velocity components	c Cold wall
x, y horizontal and vertical Cartesian Coordinates	h Hot wall

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Introduction

MHD effect on free convection heat transfer in a C-shaped cavity has been well studied over a century. Free convection is a very promising phenomenon now a days and associated with a wide range of industrial applications. Also, free convection in enclosures is encountered in many engineering systems such as cooling of electronic components, heat transfer for electronics packaging applications, ventilation in building and fluid movement in solar energy collectors etc. In the recent past, a number of studies have been conducted to investigate the flow and heat transfer characteristics in closed cavities. Only the relevant ones are cited in this paper.

Researchers have studied the concept from various perspectives. For example, Aminossadati et al. (2005) examined the effects of orientation of an inclined enclosure on laminar natural convection. Biserani et al. (2007) used Bejan's constructal theory to optimize the geometry of H-shaped cavity that intrudes into a solid conducted wall. They optimized other cavities namely, C-shaped and T-shaped cavities and found that H-shaped is superior in thermal performance. Munshi et al. (2015) numerically investigated a numerical study of free convection in a square enclosure with non-uniformly heated bottom wall and square shape heated block. Krakov et al. (2005) investigated numerically and experimentally effects of a uniform magnetic field on natural convection in a cubic enclosure. They found that a set of numerous convective structures exist in the cube. Kandaswamy et al. (2008) studied numerically magneto hydrodynamic natural convection in a square cavity with partially thermally active side walls. They considered nine different combinations of the relative positions of the active portions. Their results showed that when the active portions are located at the middle of the side walls, maximum rate of heat transfer occurs. Moreover, they found that the average Nusselt number decreases with the increase of Hartman number and increases with an increase of Grashof number. Mahmud (2004) investigated the magneto hydrodynamic free convection and entropy generation for a square enclosure at low Hartman numbers. They found that the fluid velocity is reduced with increasing value of the Hartman number. Very recently Mohmoodi et al. (2011) investigated numerically magneto hydrodynamic free convection in a square cavity with hot left wall, cold top wall and insulated right and bottom wall. They found that a clockwise primary eddy is formed inside the cavity regardless the Rayleigh number and the Hartman number. Also they found that the magnetic field decreases the intensity of free convection and flow velocity.

Nithyadevi et al. (2009) using a numerical simulation investigated effect of time periodic boundary conditions on magneto hydrodynamic natural convection in a square cavity with partially heated and cooled side walls. They found that the flow and the heat transfer rate in the cavity are affected by the sinusoidal temperature profile and by the magnetic field at lower values of Grashof number. Moreover, they found that the maximum rate of

heat transfer occurs for the active portions located at the middle of the side walls. Oztop et al. (2009) investigate numerically the magneto hydrodynamic free convection in non-isothermally heated square enclosure. They observed that the heat transfer decreases with increasing Hartman number and decreasing amplitude of sinusoidal function. Mahmoodi (2011) studied free convection in L-shaped cavity filled with Cu-water nanofluid. Mahmoodi and Hashemi (2012) studied the C-shaped cavity filled with a nanofluid. Cho et al. (2012) investigated the natural convection enhancement of Al_2O_3-water nanofluid in a U-shaped cavity. Pirmohammadi et al. (2009) studied steady laminar free convection flow in presence of a magnetic field in an enclosure heated from left and cooled from right. The Hartmann number increases with the increase of streamline. In a numerical study Ruddraiah et al. (1995) investigated the effect of a transverse magnetic field on the free convection heat transfer and fluid flow in a differentially heated rectangular with isothermal side walls and adiabatic horizontal walls. Their results showed that a circulating flow is formed with a relatively weak magnetic field. Moreover, they found that with increasing the magnetic field the convective heat transfer decreases. Mansour et al. (2014) also investigated the natural convection inside U-shaped cavity filled with Cu-water nanofluid but they termed their cavity as C-shaped. Mojumder et al. (2015) studied the natural convection in C-shaped cavity filled with Cobalt-Kerosene ferrofluid under the effect of externally applied magnetic field. Wang et al. (2007) reported results of a numerical study on magneto hydrodynamic natural convection in a porous media filled square cavity. They used the Brinkman-Forchheimer extended Darcy model to solve the momentum equations, and the local thermal non-equilibrium (LTNE) models to solve energy equations for fluid and solid. They found that both the magnetic force and the inclination angle have significant effect on the flow field and heat transfer in porous medium.

On the basis of the literature review, it appears that very little work was reported on the MHD effect on free convection heat transfer in a C-shaped cavity. Thus, the obtained numerical results of the present problem are presented graphically in terms of streamlines, isotherms, velocity, dimensionless temperature and local Nusselt number for different Rayleigh numbers and Hartmann numbers.

Physical Configuration

The physical models under consideration numerical simulation of MHD effect on free convection heat transfer in a C-shaped cavity is shown in Figure 1. The gravity acts in the negative y direction. The uniform external magnetic field of constant strength B_0 is applied in the x direction. The top, left and bottom walls are maintained at heated temperature T_h. The right internal walls are maintained at cold temperature T_c whereas the remaining parts are kept adiabatic. The two-dimensional C-shaped cavity has equal length and height of L. The internal walls of cavity with the length H are maintained at a relatively low temperature T_c.

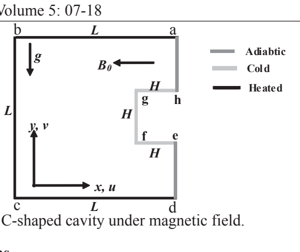


Fig. 1: Schematic of C-shaped cavity under magnetic field.

Governing Equations

The governing equations (continuity, momentum and energy equations) for laminar, steady state, two-dimensional free convection with a magnetic field in x-direction, are expressed as below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta(T-T_c)-\sigma B_0^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where u and v are the velocity components, p is pressure, ρ is the density, μ is the dynamic viscosity, β is the thermal expansion coefficient, σ is the electrical conductivity, B_0 is the magnitude of magnetic field, T is the temperature and α is the thermal diffusivity.

Boundary conditions

The boundary conditions for the present problem are specified as follows:

On walls: *ab, bc, cd* : $u = v = 0, T = T_h$

On walls: *ef, fg, gh* : $u = v = 0, T = T_c$

On walls: *de, ha* : $u = v = 0, \frac{\partial T}{\partial x}=0$

Using the following dimensionless parameters, the governing equations can be converted to the dimensionless forms:

$$X=\frac{x}{L}, Y=\frac{y}{L}, U=\frac{uL}{\alpha}, V=\frac{vL}{\alpha}, P=\frac{pL^2}{\rho\alpha^2}, \theta=\frac{T-T_c}{T_h-T_c} \tag{5}$$

The dimensionless forms of the governing equations are:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\mathrm{RaPr}\theta-\mathrm{Ha}^2\mathrm{PrV} \tag{8}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

Where Ra, Pr and Ha are the Rayleigh, Prandtl and Harmann numbers and defined as:

$$Ra=\frac{g\beta(T_h-T_c)L^3}{\alpha\nu}, \Pr=\frac{\nu}{\alpha}, Ha=\beta_0 L\sqrt{\frac{\sigma}{\rho\nu}} \tag{10}$$

where ν is the kinematic viscosity. The effect of magnetic field into the momentum equation is introduced through the Lorentz force term, $\vec{J}x\vec{B}$ that is reduced to $-\sigma B_0 v^2$as shown by Mahmoodi and Talea'pour (2011).

Dimensionless boundary conditions

On walls: *ab, bc, cd* : $U = V = 0, \theta = 1$

On walls: *ef, fg, gh* : $U = V = 0, \theta = 0$

On walls: *de, ha* : $U = V = 0, \frac{\partial \theta}{\partial X}=0$

To computation of the rate of heat transfer, Nusselt number along the hot wall of the enclosure is used that is as follows:

$$Nu_{local}=-\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} \tag{11}$$

The average Nusselt number of the hot wall is obtained as follows:

$$Nu=\int_0^1 Nu_{local}dX \tag{12}$$

The dimensionless stream function is defined as, $U=\frac{\partial \psi}{\partial Y}, V=-\frac{\partial \psi}{\partial X}$

Mesh Generation

In finite element method, the mesh generation is the technique to subdivide a domain into a set of subdomains, called finite elements, control volume etc. The discreate locations are defined by the numerical grid, at which the variables are to be calculated. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising the various fields of engineering. Fig.2 displays the finite element mesh of the present physical domain.

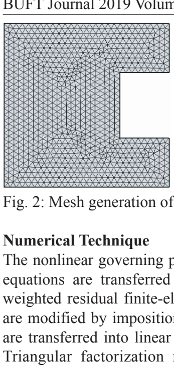


Fig. 2: Mesh generation of the C-shaped cavity

Numerical Technique

The nonlinear governing partial differential equations, i.e., mass, momentum and energy equations are transferred into a system of integral equations by using the Galerkin weighted residual finite-element method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations with the aid of Newton's method. Lastly, Triangular factorization method is applied for solving those linear equations. For numerical computation and post processing, the software COMSOL Multiphysics is used. Table 1 shows a comparison between the average Nusselt numbers obtained by the present code with the results of Pirmohammadi et al. (2009) for different Rayleigh and Hartman numbers. As can be observed from the table, very good agreements exist between the two results.

Table 1: Comparison between the average Nusselt numbers of present study and those of Pirmohammadi et al. (2009).

Ra	Ha	*Nu*	
		Pirmahammadi et al. [2009]	Present study
10^4	0	2.29	2.312
	25	1.06	1.091
	100	1.02	1.102
10^5	0	4.62	4.4380
	25	3.51	3.5012
	100	1.37	1.343

Results and Discussion

MHD effects on free convection heat transfer in a C-shaped cavity are investigated numerically. The results are gathered by inspecting the effects of Rayleigh number $10^3 \leq Ra \leq 10^6$ and Hartmann number $0 \leq Ha \leq 10^2$ on the flow are presented in the following subsections.

Effect of Rayleigh number on free convection

Streamlines and isotherms for different values of Rayleigh numbers $Ra = 10^3$- 10^6 while

$Ha = 0$ and $Pr = 0.71$ are presented in Fig. 3 to understand the effect of Rayleigh number on flow field and temperature distribution. At $Ra = 10^3$ and in the absence of the magnetic field ($Ha = 0$) one circulation cell are formed centre of the cavity shown in Fig. 3(a). As Ra increases to 10^4, the effect of free convection increases and the strength of both primary and secondary circulation increases. For higher Rayleigh numbers two or more circulation cell is formed inside the cavity and also flow strength increases are shown in Fig. 3(a). Conduction dominant heat transfer is observed from the isotherms in Fig. 3(b). As seen from the figure, isotherms are concentrates near the top and bottom walls. For higher Rayleigh number, the isotherms are bending right and bottomwalls which means increasing heat transfer through convection. In Fig. 4(a) variation of local Nusselt number along the bottom wall for different Rayleigh number with $Ha = 0$ and $Pr = 0.71$. Maximum and minimum shape curve here and Local Nusselt number increases as increasing Rayleigh number.

Fig. 4(b) shows that variations of the vertical velocity components along the horizontal wall for different Rayleigh number with $Ha = 0$ and $Pr = 0.71$. As seen from this figure, lower value of Rayleigh number has less significant change but higher value of Rayleigh number has more significant change. Fig.4(c) represents the dimensionless temperature profile along the bottom wall for different Rayleigh numbers with $Pr = 0.71$ and $Ha = 0$. As seen from the figure, maximum value of temperature decreases with increasing Rayleigh number. For lower Rayleigh number, the value of temperature has larger change but for higher Rayleigh number the value of temperature has smaller change.

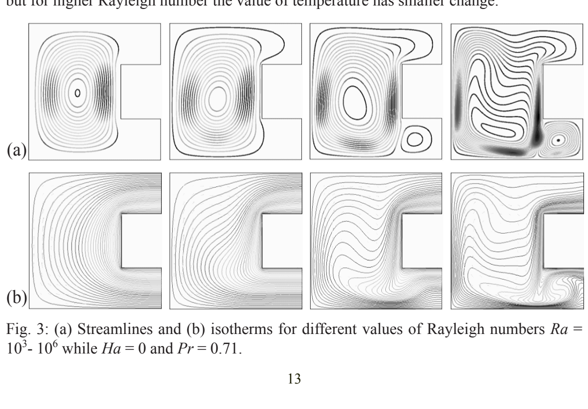


Fig. 3: (a) Streamlines and (b) isotherms for different values of Rayleigh numbers $Ra = 10^3$- 10^6 while $Ha = 0$ and $Pr = 0.71$.

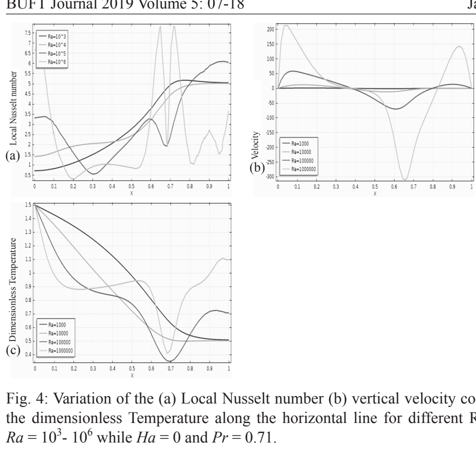
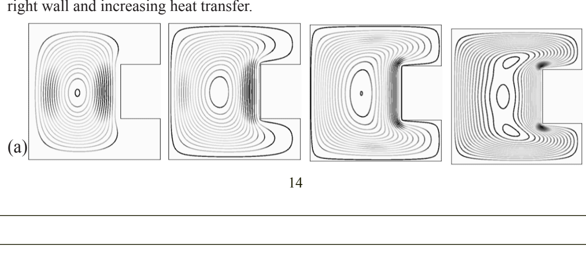


Fig. 4: Variation of the (a) Local Nusselt number (b) vertical velocity component and (c) the dimensionless Temperature along the horizontal line for different Rayleigh number $Ra = 10^3$- 10^6 while $Ha = 0$ and $Pr = 0.71$.

Effect of Hartmann number on free convection

The effect of the applied magnetic field is studied by varying Hartmann number ($Ha = 0$-10^2) at $Ra = 10^5$ and $Pr = 0.71$. The streamlines patterns, which are presented in figure 5(a). For lower value of Hartmann number, it can be found that strength of the buoyancy inside the cavity is significant and more fluid rise from the centre of the cavity. As Ha increases, the strength of the buoyancy increases and two or more circulation cells inside the cavity. At Rayleigh number $Ra = 10^5$ and $Ha = 0$-10^2 the isotherms illustrate a pure conduction heat transfer. As can be seen from the Figure 5(b) the isotherms line bending middle of the C-shaped cavity. While Ha increases isotherms lines are bending near the right wall and increasing heat transfer.



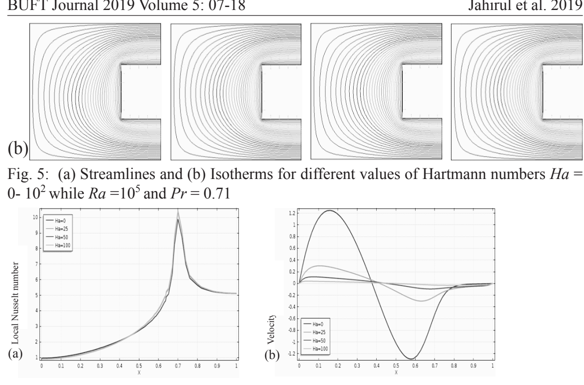


Fig. 5: (a) Streamlines and (b) Isotherms for different values of Hartmann numbers Ha = 0- 10^2 while $Ra = 10^5$ and $Pr = 0.71$

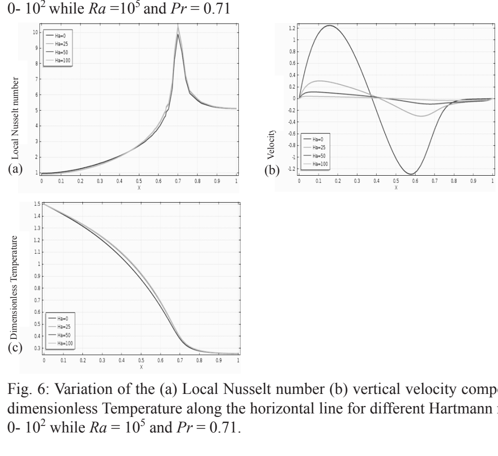


Fig. 6: Variation of the (a) Local Nusselt number (b) vertical velocity component and (c) dimensionless Temperature along the horizontal line for different Hartmann number Ha = 0- 10^2 while $Ra = 10^5$ and $Pr = 0.71$.

Figure 6(a) shows the effect of Local Nusselt number along the horizontal wall for different Hartmann number Ha = 0- 10^2 while $Ra = 10^5$ and $Pr = 0.71$. As seen from this figure, minimum and maximum shape curve here. When the value $X< 0.5$ minimum shape curves are found and $X > 0.5$ maximum shape curves found, also Local Nusselt number increasing for higher Hartmann number. Figure 6(b) represents the variations of the vertical velocity components along the horizontal wall for different Hartmann number with $Ra = 10^5$ and $Pr = 0.71$. It can be seen from the figure that the higher Hartmann number value of velocity has larger change. Fig. 6(c) shows the effect of dimensionless temperature along the bottom wall for different Hartmann numbers with $Pr = 0.71$ and $Ra = 10^5$. As can be seen from the Fig. 6(c), increase in Hartmann number motivates the

value of temperature decreases. Also dimensionless temperature decreases clearly depends on the value of Hartmann number. It is found that free convection heat transfer decrease with increases in temperature via increasing the Hartmann number. Therefore, at high Hartmann numbers, a relatively stronger magnetic field needed to decrease the rate of heat transfer. Variation of average Nusselt number versus Rayleigh number for different values of Hartmann number are shown in Fig. 7. It is also evident from this figure that, the lower value of Hartmann number average Nusselt number has more significant change but higher value of Hartmann number has less significant change. A variation of Nu_{av} between lowest value and upper value of considering parameters is presented here. In case of Rayleigh number, average Nusselt number increases 356.57% when $Ha = 0$ and $Pr = 0.71$. Again the case of Hartmann number, average Nusselt number decreases 24.01% when $Ra = 10^6$ and $Pr = 0.71$.

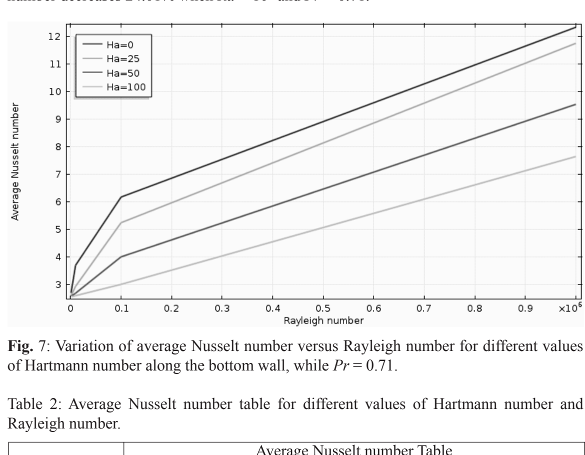


Fig. 7: Variation of average Nusselt number versus Rayleigh number for different values of Hartmann number along the bottom wall, while $Pr = 0.71$.

Table 2: Average Nusselt number table for different values of Hartmann number and Rayleigh number.

Rayleigh number	Average Nusselt number Table			
	Hartmann number			
	$Ha = 0$	$Ha = 25$	$Ha = 50$	$Ha = 100$
$Ra = 10^3$	2.70402	2.59997	2.57678	2.5679
$Ra = 10^4$	3.70937	2.94695	2.69686	2.60538
$Ra = 10^5$	6.18431	5.25383	4.61542	3.01718
$Ra = 10^6$	12.34565	11.76393	9.54944	7.65541

Conclusion

The current investigation addresses MHD free convection in a C-shaped cavity. The variations of relevant parameters in the present study are Hartmann number, Rayleigh number and Prandtl number. The effects of variation of the mentioned parameters were used on the distribution in terms of streamlines, isotherms, velocity and temperature. Good distributions were shown for different Rayleigh numbers at different Hartmann numbers. For all cases considered, two or more counter rotating eddies were formed inside the cavity regardless the Rayleigh and the Hartmann numbers. The obtained results showed the heat transfer mechanisms, temperature distribution and the flow characteristics for different Rayleigh number. Therefore, we conclude that a strong MHD is needed to compare the lower Rayleigh number with increase the buoyancy force. Moreover the significant suppression of the convection current in the cavity is due to increase of Hartmann number. Better heat transfer rate is achieved for $Ra = 10^6$ which results the exported convective heat transfer inside the C-shaped cavity.

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